Deep Embeddings of Temporal Logics in HOL

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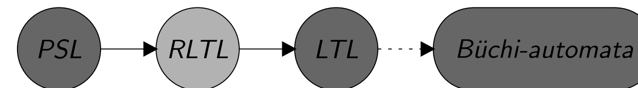
Original situation

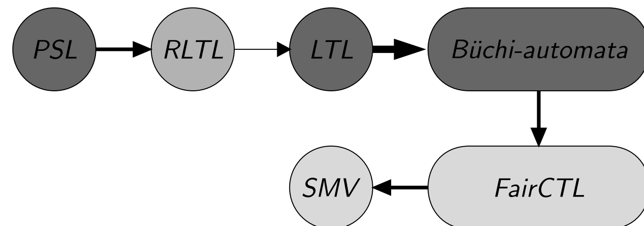
- in my diploma thesis I formally validated a translation of a significant subset of *PSL* to generalised Büchi automata
- the emptiness problem of generalised Büchi automata remained



Goal

Create an automatic tool inside *HOL* to model check *PSL* formulae!

Work done



- automatise the translation of *LTL* to Büchi-automata
- optimise the translation of *PSL* to *RLTL*
- create an interface to SMV

Work done II

- deep embeddings of important formalisms for model checking:
  - propositional logic
  - *LTL*
  - *RLTL*
  - *CTL*, *FairCTL*, CTL^*
  - kripke structures, symbolically and explicitly represented
  - automaton formulae, i. e. symbolically represented nondeterministic and universal ω-automata
  - explicitly represented nondeterministic and universal ω-automata
  - alternating ω-automata
- application: tool for IBM

Overview

1. Introduction
2. Examples
3. Extension of the Simplifier
4. Application: Tool for IBM
5. Conclusions

Translation of *LTL* to Automata

LTL formula

$$(XX\ a) \wedge G(\neg a)$$

At the next but one point of time *a* holds and a never holds!

```
> val ltl =
    ‘‘LTL_AND
        (LTL_NEXT (LTL_NEXT (LTL_PROP (P_PROP a))),
         LTL_ALWAYS (LTL_NOT (LTL_PROP (P_PROP a))))‘‘;
```


Translation to Generalized Büchi-Automata

```
- ltl2omega true true ltl;

> val it =
     []
    |- !sv.
         IS_ELEMENT_ITERATOR sv 3 {a} ==>
         !i.
           LTL_SEM i
             (LTL_AND
                (LTL_NEXT (LTL_NEXT (LTL_PROP (P_PROP a))),
                 LTL_ALWAYS (LTL_NOT (LTL_PROP (P_PROP a))))) =
           A_SEM i
             (A_NDET
                (symbolic_semi_automaton (\s. ?n. n < 3 /\ (s = sv n)) (P_BIGAND [])
                   (XP_BIGAND
                      [XP_EQUIV (XP_PROP (sv 1),XP_NEXT (P_PROP (sv 0)));
                       XP_EQUIV (XP_PROP (sv 0),XP_NEXT (P_PROP a));
                       XP_EQUIV
                         (XP_PROP (sv 2),
                          XP_OR
                            (XP_CURRENT (P_PROP a),
                             XP_AND (XP_CURRENT (P_NOT P_FALSE),XP_NEXT_PROP (sv 2))))]),
                 A_AND
                   (A_BIGAND [],
                    ACCEPT_COND_PROP (P_AND (P_PROP (sv 1),P_NOT (P_PROP (sv 2)))))))
```


Translation to Generalized Universal Persistency Automata

```
- ltl2omega true false ltl;

> val it =
     []
    |- !sv.
         IS_ELEMENT_ITERATOR sv 3 {a} ==>
         !i.
           LTL_SEM i
             (LTL_AND
                (LTL_NEXT (LTL_NEXT (LTL_PROP (P_PROP a))),
                 LTL_ALWAYS (LTL_NOT (LTL_PROP (P_PROP a))))) =
           A_SEM i
             (A_UNIV
                (symbolic_semi_automaton (\s. ?n. n < 3 /\ (s = sv n)) (P_BIGAND [])
                   (XP_BIGAND
                      [XP_EQUIV (XP_PROP (sv 1),XP_NEXT (P_PROP (sv 0)));
                       XP_EQUIV (XP_PROP (sv 0),XP_NEXT (P_PROP a));
                       XP_EQUIV
                         (XP_PROP (sv 2),
                          XP_OR
                            (XP_CURRENT (P_PROP a),
                             XP_AND (XP_CURRENT (P_NOT P_FALSE),XP_NEXT_PROP (sv 2))))]),
                 A_IMPL
                   (A_BIGAND [ACCEPT_COND_GF (P_IMPL (P_PROP (sv 2),P_PROP a))],
                    ACCEPT_COND_PROP (P_AND (P_PROP (sv 1),P_NOT (P_PROP (sv 2)))))))
```

Exploiting Termsharing

Examples of a counter with $n+1$ bits (kindly provided by Kristin Rozier).

	fast		termsharing	
1	3.9 s	24 state vars	11.7 s	13 state vars
2	5.1 s	34 state vars	14.6 s	17 state vars
3	6.5 s	44 state vars	18.4 s	21 state vars
4	7.9 s	54 state vars	23.1 s	25 state vars
5	9.5 s	64 state vars	28.8 s	29 state vars
...				
10	18.8 s	114 state vars	76.7 s	49 state vars
15	30.4 s	164 state vars	167.8 s	69 state vars
20	43.9 s	214 state vars	315.3 s	89 state vars
25	59.1 s	264 state vars	533.1 s	109 state vars

Translation to Fair Emptiness Check I

```
- ltl_contradiction2ks_fair_emptyness true ltl;

> val it =
     []
    |- !sv.
         IS_ELEMENT_ITERATOR sv 3 {a} ==>
         (LTL_IS_CONTRADICTION
            (LTL_AND
               (LTL_NEXT (LTL_NEXT (LTL_PROP (P_PROP a))),
                LTL_ALWAYS (LTL_NOT (LTL_PROP (P_PROP a))))) =
          IS_EMPTY_FAIR_SYMBOLIC_KRIPKE_STRUCTURE
            (symbolic_kripke_structure (P_AND (P_PROP (sv 1),P_NOT (P_PROP (sv 2))))
               (XP_AND
                  (XP_EQUIV (XP_PROP (sv 2),XP_OR (XP_PROP a,XP_NEXT_PROP (sv 2))),
                   XP_AND
                     (XP_EQUIV (XP_PROP (sv 1),XP_NEXT_PROP (sv 0)),
                      XP_EQUIV (XP_PROP (sv 0),XP_NEXT_PROP a))))) []) : thm
```


Translation to Fair Emptiness Check II

```
- ltl_contradiction2ks_fair_emptyness___num 1 true ltl;

> val it =
    ( []
     |- ALL_DISTINCT [a] ==>
        (LTL_IS_CONTRADICTION
           (LTL_AND
              (LTL_NEXT (LTL_NEXT (LTL_PROP (P_PROP a))),
               LTL_ALWAYS (LTL_NOT (LTL_PROP (P_PROP a))))) =
         IS_EMPTY_FAIR_SYMBOLIC_KRIPKE_STRUCTURE
           (symbolic_kripke_structure (P_AND (P_PROP 1,P_NOT (P_PROP 2)))
              (XP_AND
                 (XP_EQUIV (XP_PROP 1,XP_NEXT_PROP 0),
                  XP_AND
                    (XP_EQUIV (XP_PROP 0,XP_NEXT_PROP 3),
                     XP_EQUIV (XP_PROP 2,XP_OR (XP_PROP 3,XP_NEXT_PROP 2)))))) []),

     [(3, ``a``)]) : thm * (int * term) list
```


Translation to Fair Emptiness Check III

```
- ltl_contradiction2ks_fair_emptyness___num 2 true ltl;

> val it =
    ( []
     |- IS_EMPTY_FAIR_SYMBOLIC_KRIPKE_STRUCTURE
          (symbolic_kripke_structure (P_AND (P_PROP 1,P_NOT (P_PROP 2)))
             (XP_AND
                (XP_EQUIV (XP_PROP 1,XP_NEXT_PROP 0),
                 XP_AND
                   (XP_EQUIV (XP_PROP 0,XP_NEXT_PROP 3),
                    XP_EQUIV (XP_PROP 2,XP_OR (XP_PROP 3,XP_NEXT_PROP 2)))))) [] ==>
        LTL_IS_CONTRADICTION
          (LTL_AND
             (LTL_NEXT (LTL_NEXT (LTL_PROP (P_PROP a))),
              LTL_ALWAYS (LTL_NOT (LTL_PROP (P_PROP a))))),

     [(3, ``a``)]) : thm * (int * term) list
```

Successful Model Checking

```
- model_check___ltl_contradiction ltl;

> val it =
    SOME []
        |- LTL_IS_CONTRADICTION
             (LTL_AND
                (LTL_NEXT (LTL_NEXT (LTL_PROP (P_PROP a))),
                 LTL_ALWAYS (LTL_NOT (LTL_PROP (P_PROP a)))))
     : thm option
```

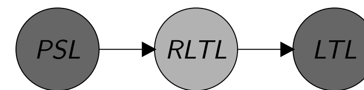

Counter Example

```
- val ltl2 =
    ``LTL_AND (LTL_NEXT (LTL_NEXT (LTL_PROP (P_PROP a))),
               LTL_ALWAYS (LTL_PROP (P_PROP a)))``

- model_check___ltl_contradiction ltl2;

Formula is not true! Consider the countermodel:
===============================================
======== A loop starts here=============
================= State0=====================
x1 = 1, x2 = 0, x3 = 1, x4 = 1
===============================================
resources used:
user time: 0 s, system time: 0 s
BDD nodes allocated: 357
Bytes allocated: 917504
BDD nodes representing transition relation: 33 + 1
reachable states: 8 (2^3) out of 16 (2^4)
===============================================
x1: sv 1, x2: sv 2, x3: sv 0, x4: a
===============================================
> val it = NONE : thm option
```


Other *LTL* Model Checking Problems

- $\forall i, t.\ i \not\models^0 ltl$
  there is no path that satisfies *ltl*, i. e. *ltl* is a contradiction
- $M \models ltl$
  the kripke structure *M* models *ltl*
- $\forall i, t.\ i \models^t ltl_1 \Leftrightarrow i \models^t ltl_2$
  ltl_1 and ltl_2 are equivalent
- $\forall i.\ i \models^0 ltl_1 \Leftrightarrow i \models^0 ltl_2$
  ltl_1 and ltl_2 are initially equivalent

Translation of *PSL* to *LTL*



Important Theorems

```
[] |- !f v b t.
        IS_INFINITE_PROPER_PATH v /\ F_CLOCK_SERE_FREE f /\ ~(t = b) /\
        PATH_PROP_FREE t v /\ PATH_PROP_FREE b v ==>

        (UF_SEM v f =
         LTL_SEM (CONVERT_PATH_PSL_LTL t b v)
            (RLTL_TO_LTL (P_PROP t) (P_PROP b) (PSL_TO_RLTL f)))


[] |- !M f. F_CLOCK_SERE_FREE f ==> (UF_KS_SEM M f = LTL_KS_SEM M (PSL_TO_LTL f))

[] |- !M f c. F_SERE_FREE f ==> (F_KS_SEM M c f = LTL_KS_SEM M (PSL_TO_LTL_CLOCK c f))
```

Problems with the Translation of *PSL* to *LTL*

1st step: *PSL* to *RLTL*

```
[] |- PSL_TO_RLTL
        (F_ALWAYS
           (F_IMPLIES
              (F_STRONG_BOOL (B_PROP aa),
               F_STRONG_NEXT_EVENT
                 (B_PROP bb,
                  F_STRONG_BEFORE (F_STRONG_BOOL (B_PROP cc),F_STRONG_BOOL (B_PROP dd)))))) =
     RLTL_NOT
        (RLTL_SUNTIL
           (RLTL_PROP P_TRUE,
            RLTL_NOT (RLTL_NOT
              (RLTL_AND
                 (RLTL_NOT (RLTL_NOT (RLTL_PROP (P_PROP aa))),
                  RLTL_NOT (RLTL_SUNTIL
                    (RLTL_PROP (P_NOT (P_PROP bb)),
                       RLTL_AND
                         (RLTL_PROP (P_PROP bb),
                          RLTL_SUNTIL
                            (RLTL_NOT (RLTL_PROP (P_PROP dd)),
                             RLTL_AND
                               (RLTL_PROP (P_PROP cc),
                                RLTL_NOT (RLTL_PROP (P_PROP dd))))))))))))
```


Problems with the Translation of *PSL* to *LTL* II

2nd step: *RLTL* to *LTL*

```
[] |- RLTL_TO_LTL P_FALSE P_FALSE ... =
       LTL_NOT (LTL_SUNTIL
         (LTL_PROP (P_OR (P_FALSE,P_AND (P_TRUE,P_NOT P_FALSE))),
          LTL_NOT (LTL_NOT (LTL_AND
             (LTL_NOT (LTL_NOT (LTL_PROP (P_OR (P_FALSE,P_AND (P_PROP aa,P_NOT P_FALSE))))),
              LTL_NOT (LTL_SUNTIL
                (LTL_PROP (P_OR (P_FALSE,P_AND (P_NOT (P_PROP bb),P_NOT P_FALSE))),
                 LTL_AND
                   (LTL_PROP (P_OR (P_FALSE,P_AND (P_PROP bb,P_NOT P_FALSE))),
                    LTL_SUNTIL
                      (LTL_NOT (LTL_PROP (P_OR (P_FALSE,P_AND (P_PROP dd,P_NOT P_FALSE)))),
                       LTL_AND
                         (LTL_PROP (P_OR (P_FALSE,P_AND (P_PROP cc,P_NOT P_FALSE))),
                          LTL_NOT
                            (LTL_PROP
                               (P_OR (P_FALSE,P_AND (P_PROP dd,P_NOT P_FALSE))))))))))))))
```

- result of the translation is unnecessary large
- simple simplification steps possible

Simplification example

3rd step: Simplification

```
[] |- LTL_EQUIVALENT
        (LTL_NOT (LTL_SUNTIL
           (LTL_PROP (P_OR (P_FALSE,P_AND (P_TRUE,P_NOT P_FALSE))),
            LTL_NOT (LTL_NOT (LTL_AND
               (LTL_NOT (LTL_NOT (LTL_PROP (P_OR (P_FALSE,P_AND (P_PROP aa,P_NOT P_FALSE))))),
                LTL_NOT (LTL_SUNTIL
                   (LTL_PROP (P_OR (P_FALSE,P_AND (P_NOT (P_PROP bb),P_NOT P_FALSE))),
                    LTL_AND
                       (LTL_PROP (P_OR (P_FALSE,P_AND (P_PROP bb,P_NOT P_FALSE))),
                        LTL_SUNTIL
                           (LTL_NOT (LTL_PROP (P_OR (P_FALSE,P_AND (P_PROP dd,P_NOT P_FALSE)))),
                            LTL_AND
                               (LTL_PROP (P_OR (P_FALSE,P_AND (P_PROP cc,P_NOT P_FALSE))),
                                LTL_NOT
                                   (LTL_PROP
                                      (P_OR (P_FALSE,P_AND (P_PROP dd,P_NOT P_FALSE)))))))))))))))
        (LTL_BEFORE (LTL_FALSE, LTL_AND
           (LTL_PROP (P_PROP aa),
            LTL_BEFORE
               (LTL_PROP (P_PROP bb),
                LTL_AND
                   (LTL_PROP (P_PROP bb),
                    LTL_SUNTIL
                       (LTL_PROP (P_NOT (P_PROP dd)),
                        LTL_PROP (P_AND (P_PROP cc,P_NOT (P_PROP dd))))))))))
```


Extension of the Simplifier

- simplification according to semantic equivalence needed
- HOL simplifier could only handle equality
- but: underlying traverser could already handle arbitrary preorders (reflexive, transitive relations)
- semantic equivalence is a congruence relation and therefore a preorder

$\Longrightarrow$ create an additional interface of the traverser to use arbitrary preorders

Extension of the Simplifier II

- `congLib` allows the usage of arbitrary preorders
- theorems have to be provided that
  - show that the relation is really a preorder
  - define congruence rules
  - define rewrite rules
- all these theorems can be combined into `congsetfrags`s and `congsets`, similar to `simpsetfrags` and `simpsets` respectively
- simplifications and tactics exist that use these `congsets`
- no conditional rewriting and no ordered rewriting available

Creating a Preorder

```
> val PROP_LOGIC_EQUIVALENT_REFL =
    [] |- !x. PROP_LOGIC_EQUIVALENT x x : thm

> val PROP_LOGIC_EQUIVALENT_TRANS =
    [] |- !x y z.
         PROP_LOGIC_EQUIVALENT x y /\ PROP_LOGIC_EQUIVALENT y z ==>
         PROP_LOGIC_EQUIVALENT x z : thm

- val prop_logic_equivalent_preorder =
    mk_preorder (PROP_LOGIC_EQUIVALENT_TRANS, PROP_LOGIC_EQUIVALENT_REFL);
> val prop_logic_equivalent_preorder =
    PREORDER((``PROP_LOGIC_EQUIVALENT'', ``prop_logic''), fn, fn) : preorder
```


Rewrite Rules

```
> val PROP_LOGIC_EQUIVALENT_rewrites =
  [] |- PROP_LOGIC_EQUIVALENT (P_NOT P_TRUE) P_FALSE /\
        PROP_LOGIC_EQUIVALENT (P_NOT P_FALSE) P_TRUE /\
        ...
        (!p. PROP_LOGIC_EQUIVALENT (P_AND (p,P_NOT p)) P_FALSE) /\
        (!p. PROP_LOGIC_EQUIVALENT (P_AND (P_NOT p,p)) P_FALSE) /\
        (!p. PROP_LOGIC_EQUIVALENT (P_OR (p,P_NOT p)) P_TRUE) /\
        (!p. PROP_LOGIC_EQUIVALENT (P_OR (P_NOT p,p)) P_TRUE) /\
        ...
        (!p1 p2. PROP_LOGIC_EQUIVALENT (P_AND (p1,P_AND (P_NOT p1,p2))) P_FALSE) /\
        ... : thm
```


Congruence Rules

```
> val PROP_LOGIC_EQUIVALENT_congs =
  [] |- (!p1 p1'.
           PROP_LOGIC_EQUIVALENT p1 p1' ==>
           PROP_LOGIC_EQUIVALENT (P_NOT p1) (P_NOT p1')) /\
        ...
        (!p1 p2 p1' p2'.
           PROP_LOGIC_EQUIVALENT p1 p1' ==>
           PROP_LOGIC_EQUIVALENT p2 p2' ==>
           PROP_LOGIC_EQUIVALENT (P_AND (p1,p2)) (P_AND (p1',p2'))) /\
        (!p1 p2 p1' p2'.
           PROP_LOGIC_EQUIVALENT p1 p1' ==>
           PROP_LOGIC_EQUIVALENT p2 p2' ==>
           PROP_LOGIC_EQUIVALENT (P_OR (p1,p2)) (P_OR (p1',p2'))) /\
        ...
        (!p p' s s'.
           PROP_LOGIC_EQUIVALENT p p' ==>
           (s = s') ==>
           (P_SEM s p = P_SEM s' p')) /\
        (!p1 p1'.
           PROP_LOGIC_EQUIVALENT p1 p1' ==>
           (P_IS_TAUTOLOGY p1 = P_IS_TAUTOLOGY p1')) /\
        ... : thm
```

Creating a Congset

```
- val prop_logic_CS = CSFRAG
   {rewrs  = [PROP_LOGIC_EQUIVALENT_rewrites],
    relations = [prop_logic_equivalent_preorder],
    dprocs = [],
    congs  = [PROP_LOGIC_EQUIVALENT_congs];}
> val prop_logic_CS =
    CSFRAG{...} : congsetfrag


- val prop_logic_nnf_CS = add_csfrag_rewrites prop_logic_CS
    [PROP_LOGIC_EQUIVALENT_nnf_rewrites];
- val prop_logic_dnf_CS = add_csfrag_rewrites prop_logic_nnf_CS
    [PROP_LOGIC_EQUIVALENT_dnf_rewrites, P_EQUIV_def, P_IMPL_def, P_COND_def];

- val prop_logic_cs = mk_congset [prop_logic_CS];
> val prop_logic_cs = <congset> : congset

- val prop_logic_nnf_cs = mk_congset [prop_logic_nnf_CS];
- val prop_logic_dnf_cs = mk_congset [prop_logic_dnf_CS];
```


Usage Examples

```
- CONGRUENCE_SIMP_CONV ``PROP_LOGIC_EQUIVALENT`` prop_logic_cs std_ss []
    ``P_AND(P_NOT(P_NOT y), x)``;
> val it =
     [] |- PROP_LOGIC_EQUIVALENT (P_AND (P_NOT (P_NOT y),x)) (P_AND (y,x)) : thm


- CONGRUENCE_EQ_SIMP_CONV prop_logic_cs std_ss []
    ``P_AND(P_PROP (3+1), P_PROP(2+2))``;
> val it =
     [] |- P_AND (P_PROP (3 + 1),P_PROP (2 + 2)) = P_AND (P_PROP 4,P_PROP 4) : thm


- CONGRUENCE_SIMP_CONV ``PROP_LOGIC_EQUIVALENT`` prop_logic_cs std_ss []
    ``P_AND(P_PROP (3+1), P_PROP(2+2))``;
> val it =
     [] |- PROP_LOGIC_EQUIVALENT (P_AND (P_PROP (3 + 1),P_PROP (2 + 2))) (P_PROP 4) : thm


- CONGRUENCE_EQ_SIMP_CONV prop_logic_cs std_ss []
    ``P_SEM s (P_AND(P_OR(b, b), a))``;
> val it =
     [] |- P_SEM s (P_AND (P_OR (b,b),a)) = P_SEM s (P_AND (b,a)) : thm
```


Problem description

- to handle a PSL formula f with IBM's model checker RuleBase, it is translated to
  - a satellite automaton A
  - a CTL formula of the form AG p
- prove, that this translation is correct for concrete examples, i. e. given A, p, f prove

$$\forall M.\ M \,||\, A \models_{\mathrm{CTL}} \mathrm{AG\ p} \iff \mathrm{M} \models_{\mathrm{PSL}} \mathrm{f}$$

- the combination of A and AG p can be considered as a universal safety automaton, i. e.

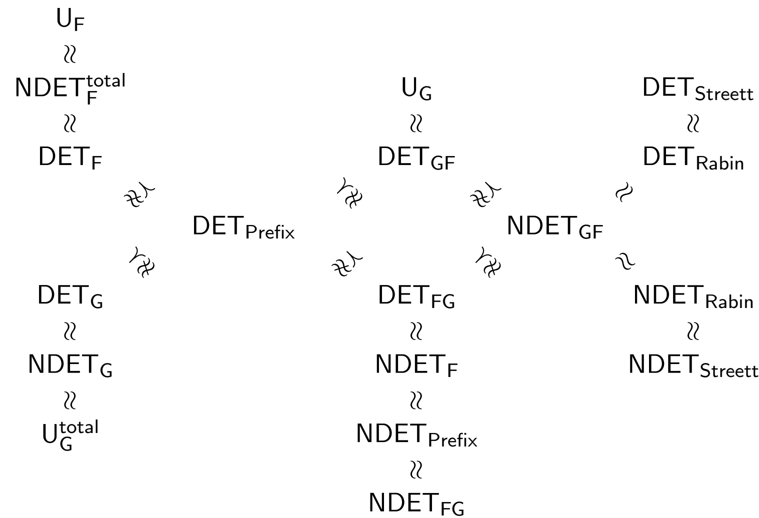
$$\forall M.\ M \,||\, A \models_{\mathrm{CTL}} \mathrm{AG\ p} \iff \mathrm{M} \models \mathcal{A}_{\forall}(\mathrm{A}, \mathrm{G\,p})$$

Used Method

$$\begin{array}{lcll}
\forall M.\ M \models \mathcal{A}_{\forall}(A, \mathrm{G}\,p) & \iff & M \models_{\mathrm{PSL}} f & \textit{iff} \\
\forall M.\ M \models \mathcal{A}_{\forall}(A, \mathrm{G}\,p) & \iff & M \models_{\mathrm{LTL}} \mathrm{PSL_TO_LTL}(f) & \textit{iff} \\
\forall i.\ i \models \mathcal{A}_{\forall}(A, \mathrm{G}\,p) & \iff & i \models_{\mathrm{LTL}} \mathrm{PSL_TO_LTL}(f) & \textit{iff} \\
\forall i.\ i \models \mathcal{A}_{\mathrm{det}}(A_{\mathrm{DET}}, \mathrm{GF}\,p_{\mathrm{DET}}) & \iff & i \models_{\mathrm{LTL}} \mathrm{PSL_TO_LTL}(f) & \textit{iff} \\
\end{array}$$

$$A_{\mathrm{DET}} \models_{\mathrm{LTL}} \mathrm{GF}\,p_{\mathrm{DET}} \leftrightarrow \mathrm{PSL_TO_LTL}(f)$$

- $A_{\mathrm{DET}} \models_{\mathrm{LTL}} \mathrm{GF}\,p_{\mathrm{DET}} \leftrightarrow \mathrm{PSL_TO_LTL}(f)$ can be handled using the presented tools
- problem: determinisation of $\mathcal{A}_{\forall}(A, \mathrm{G}\,p)$



- universal safety automata can express fairness properties
- thus, determinisation of arbitrary universal safety automata results in deterministic Büchi automata
- determinisation of arbitrary universal safety automata needs breakpoints construction
- total universal safety automata are strictly less expressive
- "normal" Rabin-Scott Subset Construction and deterministic safety automata sufficient
- luckily all satellite automata used are total, but the developed tool can also handle non total automata

Application

- exploiting totality, the developed tool is able to check most examples provided by IBM in less than 5 minutes
- these examples contain some non toy examples
- IBM is interested in using this tool for verifying the current method and for debugging new developed features
- a bug in IBM's translation has been found

Conclusions

- deep embeddings of important temporal logics provided
- fully automatic translation of *LTL* and a significant subset of *PSL* to ω-automata
- model checking of these formalisms using *SMV* possible
- side effect: additional interface to the HOL simplifier, that might be useful in general
- these deep embeddings and translations can be used for an application, IBM is interested in

Possible Future Work

- do more examples with the developed tool
- create an input parser
- implement some details more efficiently

- extend the subset of *PSL* that can be translated to full *PSL*